Computation of Fixed Points for Nonexpanding Functions

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In 1979 Leonid Khachiyan introduced the exterior ellipsoid algorithm (EEA) as the first method for solving linear programming problems in polynomial time. We show that this algorithm can also be applied for approximating fixed points of d-dimensional directionally nonexpanding functions with respect to the second norm. The number of iterations of EEA is at most $2d^2 \ln(1/\epsilon)$ for computing x: $||x - f(x)||_2 \leq \epsilon$. Several tests of a numerically stable implementation of EEA as well as comparisons with the Simple Iteration algorithm will be presented. In 1988 the Interior Ellipsoid Algorithm (IEA) was developed by Khachiyan, Tarasov and Erlikh. It turns out that the IEA algorithm enjoys an almost minimal number of iterations $6d \ln(1/\epsilon)$, however its numerically stable implementations are not known.

For the infinity norm case the ellipsoid constructions are not valid. In this case we introduce an optimal Bisection-Envelope ($d = 2$) and (non-optimal) Recursive bisection algorithms ($d > 2$). We present several numerical tests of those algorithms and formulate some open problems.

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